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Abstract

The aim of this paper is to explore under which conditions a representative agent (RA) model is able to correctly approximate the output of a more realistic model based on the "true" assumption of many interacting agents. The starting point is the widespread Keynesian cross diagram, which is compared to an extended versions that explicitly considers a multiplicity of interacting households and firms, and collapses into the original model when the number of agents is one per type. Results show that the RA Keynesian cross diagram model is not a good approximation of the extended model when (i) the network structure of the economy is not symmetric enough, e.g. firms have different sizes, or (ii) the rationality of agents is not high enough. When income inequality is considered, through the introduction of capitalists, the representative agent model is no more a good approximation, even if the agents are rational. A fiscal policy that targets income redistribution improves the prediction of the RA model. In general, all features that increase overall rationality in the economy and decrease its heterogeneity, tend to improve the performance of the RA approximation.

Key words: macroeconomics; rationality; inequality; Keynesian cross-diagram; representative agent; agent-based models; networks; simulation; complex adaptive systems
JEL classification: E00; E12; C63

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All happy families are alike; each unhappy family is unhappy in its own way.

Leo Tolstoy

Introduction

If it was possible to describe the world by means of a representative family, Tolstoy would never have been the great author we know. The meaningful content, in a literary work, depends on the fact that many diverse families exist, and without taking into account this diversity it would not be possible to create good literature. In a sense, there is no interesting literary dynamics going on in the description of the representative family.

A similar question can be posed in economics: is it possible to create good economics by relying on the assumption of a representative agent? The aim of this paper is to provide some useful elements to outline a possible answer. The problem could be reformulated in the following, more precise, terms: can we measure the error that we make, in predicting aggregate outcomes, by approximating a multiplicity of agents (that is what is given in the real world) with the hypothesis of a representative agent?

I use the word "multiplicity", which indicates the property of being multiple, instead of the more commonly used "heterogeneity", which indicates the property of being diverse, because it is more general and more relevant to the paper. Actually, multiplicity can exist without heterogeneity but not the other way around: hydrogen atoms or perfect gas molecules are endowed with multiplicity but not with heterogeneity. In the case of human beings, which can be considered as the elementary particle in economics, multiplicity can be distinguished by heterogeneity only in the abstract[1], as two identical human beings do not exist. Atoms and molecules, even if identical in the internal structure, can still undergo heterogeneous interactions, determined by the network configuration, which can lead to different aggregate outcomes. In a similar way, humans can be identical (in the abstract) and still generate different outcomes because of their diverse interactions.

The network structure that generates an aggregate "observable" is a good candidate to determine whether a model based on the representative agent hypotheses is able to successfully describe that "observable". Intuitively, if the network structure is very irregular, or complex, the probability that a representative agent model could replicate the underlying dynamics is low. On the other hand, if the network is simple, or symmetric, representative agent models should perform well, because the specific role of the network in determining the aggregate observable is limited.[2] According to this vision, the more irregular a network structure, the lower the predictive power of the representative agent, or, in other words, once the symmetry of the system is broken, the representative agent paradigm becomes weaker. This argument resembles the discussion about the "broken symmetry" in physics by Anderson [1972], where he claims that "the more the elementary particle physicists tell us about the nature of the fundamental laws the less relevance they seem to have to the very real problems of the rest of science" and that "the behavior of large and complex aggregates of elementary particles is not to be understood in terms of a simple extrapolation of the properties of a few particles". Adapting these considerations to the economic science, one faces at least two additional complications: (i) there are no fundamental laws for the elementary particle in economics (i.e. humans), (ii) these elementary particles are able to adapt their behavior to changes in the symmetry of the system.

[1] I emphasize this because the models presented in the paper are abstract, allowing therefore the distinction of the two properties.

[2] I remark that the network structure of the representative agent is the simplest, as shown later in the paper.

This last characteristic has often been considered as the essential attribute of the human being[3], and has been usually called "rationality", also by economists. In our context, rationality might also play an important role, as higher rationality would probably lead to a higher capacity of adapting to more complex systems, counterbalancing at the individual or social level, the asymmetries of a network. According to this perspective, the rationality hypothesis implies that agents are intelligent enough to smooth out the complexity given by the network structure, and that, therefore, we do not have to care too much about it. In this case, we are allowed to ignore the broken symmetry implications.

In fact, the "representative agent" and the "rationality" hypotheses often coexist in general equilibrium models,[4] and Arrow [1986] talks of the two as "auxiliary assumptions". A representative agent can actually represent a multiplicity of humans only if they are rational enough to take always the best decisions. In the context of general equilibrium models, Akerlof and Yellen [1985a,b] find that small deviations from rationality can yield significant differences in the economic equilibrium, depending on the fraction of maximising agents, which can be seen as a sort of heterogeneity measure.

This paper introduces the concept of rationality in a very different way, as there is no optimization in the Keynesian cross diagram, but still finds that a lack of rationality in a model with multiple agents produces strong deviations from the representative agent equilibrium. The main reason, again, is that scarcely rational agents are not able to adapt to the characteristics of the economy.[5] Concerning the role of rationality in economics, see for instance Becker [1962] for a seminal introduction of the topic, Russell and Thaler [1985] for a study of the implications of irrationality, Conlisk [1996] for an interesting review on incorporating bounded rationality in economic models, and all the pioneering work of Herbert Simon in this field, e.g. Simon [1955].

The question about the limitations of the representative agent descriptions has been addressed by several authors, as Leijonhufvud [1993] or Kirman [1992]. The latter notes how in textbook models, "without any precise results on the relation between the properties of individual and aggregate demand behavior, the easiest way to proceed was simply to assume that the whole economy behaved as one individual". Kirman continues stating that "all basic production and consumption can be subsumed under the activity of one amoeba-like individual who owns the one firm and consumes what it produces".

Following Kirman's advice, I try to revive the individuals stuck together into the glutinous amoeba, giving them back their multiplicity. I start from a dynamic version of the Keynesian cross diagram, which is probably the best known macroeconomic model, and a pillar of every introductory course, and I explore the implications of removing the representative agent assumption, by considering many households and many firms interacting in the economy. This exercise, that I call "disaggregation", leads to a rich variety of scenarios where the equilibrium output can be substantially different from the one predicted by the RA hypothesis. The choice of the Keynesian cross diagram is motivated by a few reasons: (i) the model is well established; (ii) it is very simple; (iii) it is characterised by a stable equilibrium that can be easily used as a benchmark to measure the deviations from the representative agent case.

Results confirm intuition. The model based on the representative agent, which is a sort of average agent, becomes inaccurate when the network structure is more asymmetric, and when the agents are less rational, even if they are homogeneous in all their individual attributes.

[3]According to a classical philosophical perspective, initiated by Aristotle (e.g. Nicomachean Ethics I.13) and accepted by scholasticism, Kant, and many other scholars, the essence of human beings consists in "rationality", making them different from all other animals.

[4]For example, DSGE models generally treat a society of utility-maximizers as if it consisted of a single "representative" individual (see e.g. Christiano et al. [2005] and Smets and Wouters [2003]).

[5]This means, in particular (see section 2), that agents are not able to understand where to buy goods in order to avoid rationing.

When the network structure is more complex, agents are more prone to make mistakes, they get caught in the cobweb of intricate interactions, and they are not able to disentangle it. Therefore, the prediction of the representative agent model becomes wrong, as it does not take into account the bounded rationality of his parts. On the other hand, when rational agents are able to make appropriate decisions, even in a complex environment, then the prediction of the representative agent is good.

The trade-off between complexity and rationality can be also interpreted in the light of Gode and Sunder [1993], who discuss about markets as partial substitutes for individual rationality, showing that allocative efficiency derives largely from its structure, independent of traders' motivation, intelligence, or learning. In this paper the concept of allocative efficiency can be defined as the state in which individuals are able to allocate their consumption over firms in order to maximize the total output. Like in Gode and Sunder [1993], the level of individual rationality and the structure of the market (or better: of the network) both contribute to determine the "allocative efficiency", and for some particular network structures, a modest level of rationality may be sufficient to reach the optimal allocation.

Moreover, Arrow [1986] states that "the homogeneity assumption seems especially dangerous...", as "...it takes attention away from a very important aspect of the economy, namely, the effects of the distribution of income and of other individual characteristics on the working of the economy". In this respect, the simple model presented in this paper is able to investigate the impact of the homogeneity assumption on the aggregated output. Results show that aggregate demand tends to be smaller in the case of income heterogeneity because the rationing of consumption of lower income households is not compensated by an excess consumption of higher income households, thus determining a lower aggregate output. Of course, results reflect the simplicity of the model, which does not incorporate mechanisms to transfer purchasing power from high to low income households, as financial and credit markets. Some considerations on these aspects are provided when discussing results. The model shows that a redistributive tax is able to mitigate the problem, reducing the excess of private savings in the economy, and increasing aggregate demand and output.

From a methodological point of view, in order to extend the Keynesian cross diagram to a multiplicity of agents, I design an agent-based model embedded in a network structure. LeBaron and Tesfatsion [2008] argue that the Agent-based Computational Economics (ACE) methodological approach provides macroeconomists "with a tremendous flexibility to tailor the breadth and depth" of the agents in their models. In recent times, several macroeconomic studies, based on the ACE methodology, flourished in the literature[6], even if they still occupy a marginal position. Understanding the reasons of this marginal position is not within the scope of this paper[7], however, I think that a major problem is the scarce comparability between general equilibrium models and agent-based macroeconomic models, because they have different underlying structures. In this respect, this paper tries to fill the gap, comparing a simple representative agent model, like the Keynesian cross diagram, with its multi-agent extension, therefore providing a way to understand under which circumstances the two models differ, and to measure the value added of the multi-agent version in different scenarios. In the same spirit, Rahmandad and Sterman [2008] compare the outcomes of agent-based and differential equation contagion models, claiming that the results of this comparison should guide the choice of models for policy analysis. In future works, the "disaggregation" methodology presented in this paper could be

[6]A non-exhaustive list of macroeconomic agent-based models (ABMs): Delli Gatti et al. [2005], Dosi et al. [2010], Cincotti et al. [2010], Dawid et al. [2014]. A survey on the topic is in Dawid and Delli Gatti [2018].

[7]Richiardi [2017] proposes a general reflection on the present and the future of agent-based modelling, while Fagiolo and Roventini [2017] provide a critical comparison between the ABM and the DSGE approaches (the comparison is written by ABM advocates, but I am not aware of any similar work written by DSGE researchers).

extended to other models that do not explicitly consider the multiplicity of economic agents.

The paper is organized as follows. The models are presented in section 1 and results are discussed in section 2. A critical synthesis of the paper is provided in the conclusions, along with some basic policy implications and potential future developments.

1 The models

This section presents the "disaggregation" process that transforms the original Keynesian cross diagram into an extended multi-agent model, which incorporates a multiplicity of interacting agents. The baseline dynamic version of the Keynesian cross diagram, or core model, is described in section 1.1, while section 1.2 presents an intermediate step, where heterogeneity is introduced in the consumption function of households. The multi-agent model, described in section 1.3, collapses into the core model in the case of just one household and one firm: it can be therefore considered as a generalization of the Keynesian cross diagram.

1.1 The core

The so called "Keynesian cross diagram" constitutes the dominant paradigm in basic macroeconomics textbooks. The model assumes that the aggregate demand of consumption goods can be described as the sum of a constant component c_0, and another one proportional to income: $c_1 Y$. When production is lower than the aggregate demand, the excess demand drives it toward the equilibrium level, through the depletion of inventories. When production is higher than demand, the excess supply generates an accumulation of inventories and a consequent decrease in production toward the equilibrium level. The typical representation of the aggregate demand function is: $Z = C + G + I + NX$.[8] To the scope of this paper, it suffices to focus on consumption, which is the only endogenous component of demand in the basic Keynesian cross diagram.[9]

When production Y is equal to aggregate demand Z, which coincides to consumption C in our simplified model, the economy is in equilibrium and the corresponding output can be defined as: $Y^* = \frac{c_0}{1-c_1}$. If Y_0 is the initial level of income, the dynamic version of the model becomes:

$$\begin{cases} Y(t+1) = c_0 + c_1 Y(t) \\ Y(0) = Y_0 \end{cases} \tag{1}$$

In the spirit of the Keynesian cross diagram, firms adapt production to demand. In particular, firms produce at time $t+1$ an amount of goods equal to the demand they faced at time t. The globally stable solution of system 1 is

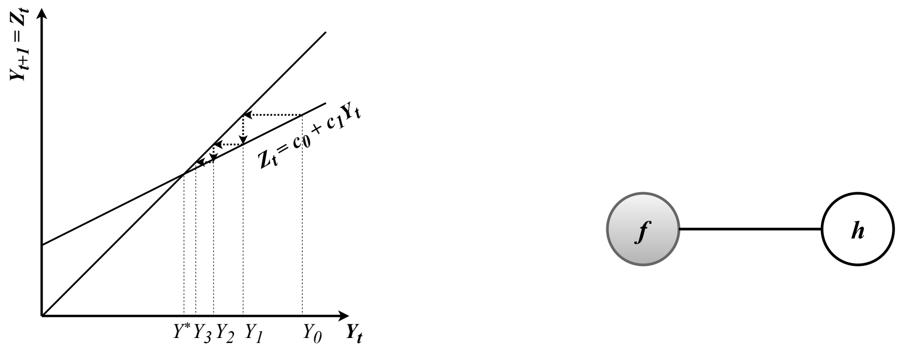
$$Y(t) = \big(Y_0 - \frac{c_0}{1-c_1}\big)c_1^t + \frac{c_0}{1-c_1}, \tag{2}$$

converging asymptotically to $Y^* = \frac{c_0}{1-c_1}$, which is the autonomous spending (c_0 in this case) multiplied by the Keynesian multiplier.

This simple dynamic model represents explicitly the convergence process toward the equilibrium of the static Keynesian cross diagram, as shown in figure 1a, where the case of initial excess supply, $Y_0 > Z(Y_0)$, is presented. Firms accumulate inventories, decrease production

[8] $C = c_0 + c_1 Y$ is consumption, G public spending, I investments, NX net exports.

[9] I am not interested here in the economic relevance of the components of aggregate demand but only in their functional role. Thus, I do not distinguish between the different exogenous components, which are eventually reduced to c_0.



(a) Dynamics.

(b) Topology

Figure 1: The Keynesian cross diagram

to Y_1, still lower than $Z(Y_1)$, thus leading to a further reduction of production, until the final equilibrium Y^* is asymptotically reached.

Figure 1b shows the plain topology of the aggregated model, where we can think of one representative household that receives an aggregate income from a representative production sector (firm), and consumes it according to its linear consumption function. The topology of the model will be further examined in section 1.3.

One of the main assumptions of this short-run model is that the price level is constant, determining *de facto* a correspondence between real and nominal variables. All the models presented in the paper preserve this feature of constant price, and therefore a substantial equivalence between real and nominal variables. Henceforth, I will conventionally call the model described in equation 1 as the Representative Agent model, or simply the RA model.

1.2 Statistical heterogeneity

Even without introducing an explicit topology in the model, it can be shown that the presence of heterogeneity weakens the "predictive power" of the representative agent core model. In other words, the equilibrium income $Y^* = \frac{c_0}{1-c_1}$ is no more valid if the propensity to consume of households is heterogeneous.

Let's assume that $\boldsymbol{c_1}$ and $\boldsymbol{c_0}$ are vectors containing the propensity to consume and the autonomous consumption, respectively, of the N households in the economy. Coherently with the adaptive rule of eq. 1, the law of motion of output is $Y(t+1) = \sum_h^N \left[c_{0h}(t) + c_{1h}(t) y_h(t)\right]$, where $y_h(t)$ is the income of household h at time t, or the h^{th} element of the vector of income $\boldsymbol{y}(t)$. It can be simply written as $Y(t+1) = C(t) = \sum_h^N c_{0h}(t) + \sum_h^N c_{1h}(t) y_h(t)$, where $C(t)$ is aggregate consumption. To stick to the original Keynesian cross diagram, let's drop the time dependence of the parameters.

The first term of the equation, the aggregate autonomous consumption $\sum_h^N c_{0h} = c_0$, can be described by the average autonomous consumption across households, multiplied by N, i.e, $c_0 = \bar{\mathbf{c_0}} N$. However, in the second term there is an interaction between income and propensity to consume. The average propensity to consume (or the representative one) is no more able to describe the aggregate consumption function. In particular, $C(t) = c_0 + \sum_h^N c_{1h} y_h(t)$, or $C(t) = c_0 + \mathbf{c_1}^\intercal \cdot \mathbf{y}(t)$, where $\mathbf{c_1}^\intercal$ is the transpose of $\mathbf{c_1}$.

Considering that the covariance between $\mathbf{c_1}$ and $\mathbf{y}(t)$ can be written as $\sigma\big(\mathbf{c_1}, \mathbf{y}(t)\big) = \frac{1}{N}\mathbf{c_1}^\intercal \cdot \mathbf{y}(t) - \bar{\mathbf{c_1}}\bar{\mathbf{y}}(t)$, which becomes $\sigma\big(\mathbf{c_1}, \mathbf{y}(t)\big) = \frac{1}{N}\mathbf{c_1}^\intercal \cdot \mathbf{y}(t) - \bar{\mathbf{c_1}}\frac{Y_t}{N}$, then the aggregate consumption of the economy can be expressed as $C(t) = \bar{\mathbf{c_0}} N + \bar{\mathbf{c_1}} Y_t + N\sigma\big(\mathbf{c_1}, \mathbf{y}(t)\big)$, and the dynamics of the model becomes:

$$\begin{cases} Y(t+1) = c_0 + c_1 Y(t) + N\sigma\big(\mathbf{c_1}, \mathbf{y}(t)\big) \\ Y(0) = Y_0 \end{cases} \tag{3}$$

The first two terms are the ones used in the representative agent (RA) model, described in eq. 1, but the third term depends on agents' heterogeneity. If the propensity to consume of households is negatively correlated with their income ($\sigma\big(\mathbf{c_1}, \mathbf{y}(t)\big) < 0$), then the RA consumption function is overestimating the correct one. If it is positively correlated with income ($\sigma\big(\mathbf{c_1}, \mathbf{y}(t)\big) > 0$), then the RA consumption function of eq. 1 is underestimating the correct one. Only if the propensity to consume is scarcely correlated with income ($\sigma\big(\mathbf{c_1}, \mathbf{y}(t)\big) \sim 0$), then the RA consumption function is close to the correct one. If we assume that the covariance is bounded, i.e., $\forall t,\ \sigma\big(\mathbf{c_1}, \mathbf{y}(t)\big) \in [-\sigma_m, \sigma_M]$, then the equilibrium output will be in the range: $Y^* \in \left[\frac{c_0 - N\sigma_m}{1-c_1}, \frac{c_0 + N\sigma_M}{1-c_1}\right]$, and the interaction between propensity to consume and income could be interpreted as a sequence of shocks to aggregate demand, which determine the final equilibrium output within that range. It is worth noting that, according to the Consumer Expenditure Surveys[10], propensity to consume is lower for higher quantiles of income, implying an overestimation of output by the RA model. Jappelli and Pistaferri [2014] confirm these findings for the Italian Survey of Household Income and Wealth.

1.3 The topology

The simple topology of figure 1b shows a representative household that consumes from a representative firm that, in turn, transfers its revenues back to the household. Representative agents might provide a good or bad approximation of reality, but for sure they are modelling artifacts that do not exist in the real world. In this section, the RA will be replaced by a multiplicity of households h and firms f, belonging to some given sets H and F, respectively. The economy will be then composed by $|H|$ households and $|F|$ firms, where the vertical bars denote the cardinality of the sets. In order to provide some symmetry to the models, the total number of households will be generally considered as a multiple of the total number of companies, i.e., $|H| = \alpha|F|$.

As "each unhappy family is unhappy in its own way", introducing a plurality of agents opens the door to many potential configurations of the model, in terms of individual attributes, topology of the network and interaction patterns. Therefore, in order to define the model, several choices have to be made to specify the single characteristics of the agents (heterogeneity definition) and the way they interact (network definition). The number of parameters will then depend on the degree of heterogeneity and on the complexity of interactions. In this paper, I tried to minimize the complexity of the agent-based model, introducing no heterogeneity in the parameters of the households and a few different network structure, mainly characterized by their level of symmetry. The rationale for designing the model has been to stick as much as possible to the original Keynesian cross diagram, in order to be able to compare the aggregated and the "disaggregated" models in a meaningful way, assessing the net effect of the introduction of a multiplicity of agents, embedded in some given topology, on the model outcomes.

The topology of the model can be represented as a dynamic, bipartite graph $G(V, E)$, where the vertex set V can be partitioned into two subsets H and F, households and firms, and every edge $e \in E$ has one end in H and one end in F. Such a partition (H, F) is called a bipartition of the graph G, while H and F are its parts. Also the edge set E can be partitioned into two subsets E_C and E_J, representing consumption and job links, respectively. An incidence function

[10] The Consumer Expenditure Surveys program provides data on expenditures, income, and demographic characteristics of consumers in the United States: `https://www.bls.gov/cex/home.htm`.

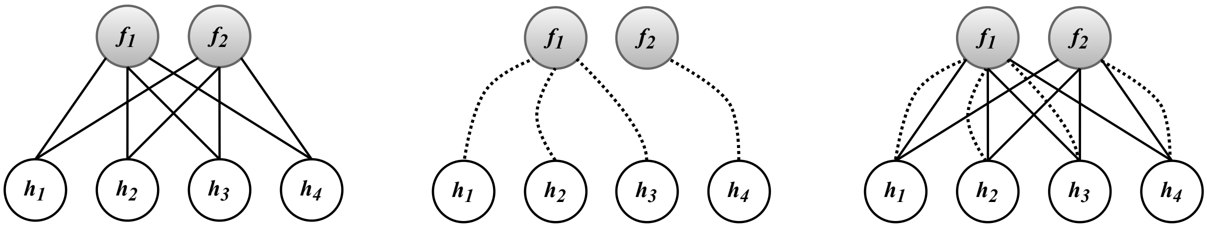


Figure 2: An example of the topology of the model with 4 households and 2 firms. The continuous lines represent consumption links, while the dotted lines are job links.

ψ associates with each edge of G a pair of vertices of H and F. In particular, if $e_C \in E_C$ is an edge, $\psi(e_C) = (h, f)$, with $h \in H$ and $f \in F$, shows the existence of a consumption link between household h and firm f. In a similar way, if $e_J \in E_J$, $\psi(e_J) = (h, f)$ shows that household h is working in firm f.

Figure 2 shows an example of the topology of the model with 4 households and two firms. Each household works in a firm and consumes from all firms. The graph is bipartite as there is no direct interaction among agents of the same category.

In the computational experiments presented in this paper, the number of households $|H|$ and firms $|F|$ is constant. Also $|E_J|$ and $\psi(e_J)$ do not change over time, meaning that a firm has always the same number of employees. The only dynamics in the topology of the graph regards $|E_C|$ and $\psi(e_C)$, which can vary over time. As it will be shown in section 1.5, households can modify their consumption links according to different levels of rationality, from a zero-intelligence scenario to more rational behaviors. For instance, if an household happens to be rationed by a firm, it will generally cut the consumption link with that firm, looking for another one with higher availability of products. In more formal terms, the dynamics in the topology of the model from time t to $t+1$ can be described as $G(t+1) = G(t) \cup E_C^+(t) \setminus E_C^-(t)$, where $E_C^+(t)$ and $E_C^-(t)$ are the subsets of edges which are added or cut in time t, respectively.

1.4 Balance sheets

Households and firms have dynamic interactions that are directly derived from the Keynesian cross diagram of equation 1. Firms distribute revenues, in form of income, to households, who spend it to buy firms' products. These two payments, from firms to households and from households to firms, flow through job links and consumption links, respectively, according to the topology of the model. These flows determine the law of motion of the balance sheet items of the agents, which are presented in table 1.

Table 1: Agents' balance sheets

Household h

Assets	**Liabilities**
w_h: wealth	e_h: equity

Firm f

Assets	**Liabilities**
s_f: inventory	e_f: equity

Household's wealth is augmented by income and depleted by consumption: $w_h(t+1) = w_h(t)+y_h(t)-c_h(t)$. At the same time, wealth constitutes a budget constraint for consumption,[11] as no debt instrument is designed in the model. Therefore, net worth e_h is the only liability in household's balance sheet.

[11] see section 1.5 for details

In a similar way, firm's inventory $s_f(t)$ is augmented by production and depleted by sales:[12] $s_f(t+1) = s_f(t) + y_f(t) - r_f(t)$. Inventories act as a buffer for firms, which allow them to sell more goods than their current production, i.e. at time t firm f can sell a maximum amount of goods equal to $s_f(t) + y_f(t)$. Households can be rationed if a firm runs out of stocks. Firms cannot take loans either, and their equity is always positive and equal to inventories. The gap between production and sales represents the deviation from the equilibrium of the economy, both at the micro level, where $\delta_f(t) = y_f(t) - r_f(t)$ is the accumulation of inventories of firm f, and at the macro level, where $\Delta(t) = Y(t) - R(t)$ is the aggregate excess supply.

1.5 Behavior and rationality

In the core model of section 1.1, the dynamics is driven by the excess demand. The representative firm reduces production in the case it faces a lack of demand, accumulating inventories, and increases it when there is excess demand, depleting inventories. The representative consumer, which generates the aggregate demand, behaves according to: $Z(t) = c_0 + c_1 Y(t)$.

When dealing with multiple agents, I design exactly the same behavioral rule, taking into account the local budget constraint. Therefore, the demand of goods of each one of the $|H|$ households in the economy at time t is

$$z_h(t) = \min\left[c_{0h} + c_{1h} y_h(t), w_h(t)\right], \tag{4}$$

where[13] $\sum_{h=1}^{|H|} c_{0h} = c_0$, $\frac{1}{|H|}\sum_{h=1}^{|H|} c_{1h} = c_1$, $\sum_{h=1}^{|H|} y_h(t) = Y(t)$, and $w_h(t)$ is the total wealth of household h at time t. Equation 4 shows that household h cannot consume more than its total wealth.

Given its consumption links set e_{Ch}, household h purchases the desired consumption goods of equation 4 in equal shares from all the connected firms. Moreover, the household is able to choose how to allocate its consumption budget over the existing firms by modifying the set e_{Ch}. For example, if household h has been rationed by firm f, it could remove the consumption link with firm f from the set e_{Ch}, and it might decide to create a new connection with another firm. This decision about consumption allocation is driven by the level of rationality of the household, which is defined in table 2. The idea is to study the impact of different levels of household's rationality on the equilibrium output of the economy. The first two levels of rationality, called "zero-intelligence", refer to households never changing their consumption links set, or doing it randomly. In both cases the rationality of households is "zero" because they do not adapt to any economic input, or information. The next level considers that households stop buying from a firm if they have been rationed by it, and that they always chose, randomly, a new firm to connect to. The last case, called "semi-rational", is like the previous one, with the difference that household h connects to a new firm only if it has not been able to consume the total desired amount $z_h(t)$; in other words, a household could permanently reduce the set of firms where it is buying, if they are able to satisfy its demand. Table 2 presents the four rationality endowments with respect to the dynamics of consumption links in a more formal way. The network is initialized with a given amount of consumption links per household, which are iteratively and randomly assigned to non-connected firms according to a uniform distribution. It is worth noting that the number of initial links does not affect the main results of the paper. Concerning job links, they are one per household, as I assume that each household can be employed by just one company.

It should be noted that the concept of households' rationality of the paper is not an arbitrary one, but derives naturally from the "disaggregation" process. If there are just one firm and one

[12] Sales are equal to revenues, and they are indicated by r_f.

[13] As explained in section 2, c_{0h} and c_{1h} are assumed equal across households in all the presented scenarios

Table 2: Households' behavioral schemes

Rationality of households[a]	**Number of initial links**[b]	**consumption links dynamics**[c] $\boldsymbol{e_{Ch}(t+1) = e_{Ch}(t) \cup e^{+}_{Ch}(t) \setminus e^{-}_{Ch}(t)}$
Zero intelligence 1 (random links)	$\frac{\lvert F\rvert}{2}$	$e^{-}_{Ch}(t) = \mathcal{R}_h[\psi^{-1}(h, F_h(t), t)]$ $e^{+}_{Ch}(t) = \mathcal{R}_h[\psi^{-1}(h, \overline{F_h(t)}, t)]$
Zero intelligence 2 (fixed links)	$\frac{\lvert F\rvert}{2}$	$e^{-}_{Ch}(t) = \emptyset$ $e^{+}_{Ch}(t) = \emptyset$
Bounded rational	$\frac{\lvert F\rvert}{2}$	$e^{-}_{Ch}(t) = \psi^{-1}(h, F_h(t), t) \mid \delta(h, f \in F_h(t), t) > 0$ $e^{+}_{Ch}(t) = \mathcal{R}_h[\psi^{-1}(h, \overline{F_h(t)}, t)]$
Semi rational	$\frac{\lvert F\rvert}{2}$	$e^{-}_{Ch}(t) = \psi^{-1}(h, F_h(t), t) \mid \delta(h, f \in F_h(t), t) > 0$ $e^{+}_{Ch}(t) = \begin{cases} \mathcal{R}_h[\psi^{-1}(h, \overline{F_h(t)}, t)] \iff \exists f \in F_h(t) \mid \delta_h(f,t) > 0 \\ \emptyset \iff \nexists f \in F_h(t) \mid \delta_h(f,t) > 0 \end{cases}$

[a] The definitions are purely qualitative and have the only aim to label the behaviors of households.
[b] These numbers of consumption links are the ones used in the simulations presented in the paper, but results are robust with respect to their variation. For example if bounded rational agents have lower initial links $|F|/4$ or zero intelligent agents have higher initial links, the results of the paper are still valid. Initially, each household assigns randomly its consumption links to $|F|/2$ different firms.
[c] Here $e_{Ch}(t)$ is the set of consumption links (edges) of household h at time t; $e^{-}_{Ch}(t)$ and $e^{+}_{Ch}(t)$ are the sets of lost links and new links at time t, respectively; $\mathcal{R}_h[A]$ is a function that returns one random element from the set A according to a uniform distribution; $F_h(t)$ is the subset of firms connected to household h at time t; ψ^{-1} is an inverse incidence function that, given an household h and a subset of firms F_x, returns all the potential edges between h and each member of F_x; $\overline{F_h(t)}$ is the complement of subset $F_h(t)$, corresponding to the firms that are not connected to h; $\delta_h(f,t)$ is the excess demand of household h with respect to firm f at time t.

household in the economy, speaking of allocation of consumption across firms has no sense. However, when many households and firms exist, there is a need to define the behaviour of the household and, consequently, to define its rationality. Therefore, introducing this specific concept of rationality is not an optional feature of the model, but a necessary step.

In line with the Keynesian cross diagram, firm f plans a production equal to past demand, which is the sum of customers' demands, i.e. households connected to the firm through a consumption link $e_{Cf}(t)$. Therefore, if $H_{Cf}(t)$ is the set of customers of firm f at time t, defined as $H_{Cf}(t) = \{h \mid h \in \psi(e_{Cf}(t))\}$, the demand faced by firm f is

$$z_f(t) = \sum_{h \in H_{Cf}(t)} z_h(t). \tag{5}$$

However, production can be also constrained by the set of available workers, which can be defined as $H_{Jf} = \{h \mid h \in \psi(e_{Jf})\}$[14]. In this respect, I assume that labor productivity β_h is homogeneous and constant over the households, which are able, when fully employed, to produce the equilibrium output

$$\beta_h = \beta = \frac{1}{|H|} \cdot \frac{c_0}{1 - c_1}, \forall h \in H \tag{6}$$

[14] I drop the time dependence here, as the employees of a firm do not change in time.

I assume that households work standard working hours, but they can also work overtime if the firm needs to produce more. Parameter $extra$ defines the maximum percentage of additional hours that households are available to work, e.g. the ones permitted by the maximum working hours law. This feature is conceptually needed in order to allow firms to temporarily face a high demand, which is in line with the demand driven spirit of the Keynesian cross diagram. It is worth noting that the value of $extra$ can be set to 0 or to larger values without affecting the main conclusions of this paper. Therefore, if firm f employs enough workers, it produces an amount of goods equal to its past demand:

$$y_f(t+1) = \min\left[z_f(t), \beta \cdot (1 + extra) \cdot |H_{Jf}|\right]. \tag{7}$$

The revenues, or sales, $r_f(t)$ of firm f are distributed as income to a subset of the households, which in general corresponds to the workers H_{Jf} of the firm.

$$\sum_{h \in H_{Jf}} y_h(t+1) = r_f(t), \tag{8}$$

For the sake of simplicity and realism, equation 8 conjectures that a household receives income only from one firm, as $\forall f \neq k$, $H_{Jf} \cap H_{Jk} = \varnothing$, with $f, k = 1, 2 \ldots |F|$.

In general, I assume that revenues are distributed to the workers of the firm in equal parts, i.e., $y_h(t+1) = r_f(t)/|H_{Jf}|$, $\forall h \in H_{Jf}$. In some scenarios this assumption will be relaxed, allowing for the unequal distribution of firm's revenues, which will be associated with the presence of shareholders in a profit-making company[15].

2 Results and discussion

This section presents a comparison between the representative agent (RA) model and the multi-agent (MA) one. In particular, I assume that the MA version of the model is the "true" one, and that the RA model is an approximation of this "true" version that ignores the multiplicity of agents. This approximation can be accurate or not, depending on the characteristics of the economy, i.e., the attributes of the agents and the topology of the network. I then define an approximation error, which measures the inaccuracy of the RA model in replicating the MA model, and I study under which conditions this error is small, meaning that the RA model is a good approximation of the MA one, or large, meaning that the RA model is not doing a good job.

Considering the MA version as the "true" model depends on the simple and direct observation of the real world, where no representative agent exist. RA models are not intended to study problems that involve direct interactions among economic agents or coordination issues but are designed to examine some central macroeconomic phenomena (see Kirman [1992] for a discussion on the topic). However, economic agents exist and interact each other, and ignoring this has a cost in terms of accuracy. This paper is a tentative to measure this cost.

Moreover, the MA model can be considered as a generalization of the RA model, since it collapses into the Keynesian cross diagram of equation 1 in the specific configuration of $|H| = |F| = 1$. Using this particular configuration (the RA one) can be a functional simplification in many cases, but in other cases it might be simply too imprecise. It is, therefore, worth investigating this trade off between using a more complex, but more realistic, MA model, or a simplified RA version. This trade-off has, on the one hand, the cost of dealing with more intricate modeling choices and lower control of results and, on the other hand, the cost of having

[15] These scenarios are presented in section 2.2.

potentially inaccurate outcomes. In order to measure this inaccuracy in different scenarios, I compute the deviation of the equilibrium output predicted by the Keynesian cross diagram from the asymptotic behavior observed in the agent-based model.

Finally, we can interpret the RA equilibrium as the best possible outcome of the economy (see eq. 6), when no frictions due to the multiplicity of agents are present. In this perspective, we can interpret the deviation of the MA output from the RA one, as the cost paid, in terms of output reduction, for the presence of many interacting agents that are not always able to coordinate properly and to take the right decisions.

The main results are presented in propositions with informal proofs. This organization allows for a better arrangement of the section, emphasising the most important findings and improving readability. The explanation of results is generally based on the observation of simulation outcomes, and not on strict inference rules (i.e., formal proofs), which are hardly applicable in this context. The aim is to provide a meaningful economic discussion and not to demonstrate mathematical theorems.

Definition 1. The representative agent equivalent, or RA equivalent, of a given multi-agent model $G(V, E)$ where household h is characterized by c_{0h} and c_{1h}, and firm f produces an initial output Y_{0f}, is defined as in equation 1 with $c_0 = \sum_{h=1}^{|H|} c_{0h}$ (the sum of all individual autonomous consumptions), $c_1 = \frac{1}{|H|}\sum_{h=1}^{|H|} c_{1h}$ (the average of individual propensities to consume) and $Y_0 = \sum_{f=1}^{|F|} Y_{0f}$ (the sum of the initial production of each firm). Moreover, the solution $Y_{RA}(t)$ of the RA equivalent model, obtained in eq. 2, is called the RA solution and the limit of this solution for $t \to \infty$, or simply $Y_{RA}(\infty)$, is called the RA equilibrium ∎

It is worth noting that $Y_{RA}(\infty) = \frac{c_0}{1-c_1}$. Moreover, if we assume that c_{0h} and c_{1h} are homogeneous over households[16], the two parameters of households can be obtained as $c_0 = |H| \cdot c_{0h}$, and $c_1 = c_{1h}$.

Definition 2. The quasi-equilibrium of a multi-agent model $Y_{MA}(\mathcal{T})$, or simply Y_{MA}, is defined as the average value, computed over the time span $\mathcal{T}$, of the aggregate output trajectory when it becomes stationary ∎

Since the asymptotic behaviour of the MA model output might not be (and in general is not) a fixed point[17], I need an approximate measure of the output, which is provided by Y_{MA}.

Definition 3. Being $Y_{RA}(\infty)$ the RA equilibrium, and Y_{MA} the quasi-equilibrium of a multi-agent model, the "deviation from the RA equilibrium" is defined as:

$$\Delta_Y = \frac{|Y_{MA} - Y_{RA}(\infty)|}{Y_{RA}(\infty)} \quad \blacksquare \tag{9}$$

I will refer to the quantity Δ_Y also as the "prediction error of the RA model", as the "true" value to predict is Y_{MA}. [18]

[16] This is the standard assumption in the paper, implying the presence of multiplicity but not of heterogeneity.

[17] The fluctuations in output are mainly due to the dynamics in the topology of the network. In the zero-intelligence 1 case of table 2, for instance, there is a continuous replacement of consumption links.

[18] Since $Y_{RA}(\infty) \geq Y_{MA}$ for every MA model, $\Delta_Y \in [0, 1]$. An alternative choice is: $\Delta_Y = \frac{|Y_{RA}(\infty) - Y_{MA}|}{Y_{MA}}$. This formulation is probably more appropriate, as the "true" value is Y_{MA}, however the (much larger) error would not be confined in the interval $[0, 1]$.

The output trajectories of the multi-agent models are obtained by running computer simulation. Unless otherwise stated, the number of households, $|H|$, used in the computational experiments is 100 and the number of firms, $|F|$, is 10. In the next sections I analyze results, focusing on the dependence of the prediction error Δ_Y on the different configurations of the multi-agent model. In particular, I will examine the effect of households' rationality and network asymmetries in the following section, and the effect of inequality in the subsequent one.

2.1 Regularity versus rationality

The RA model presented in equation 1 can be considered as the description of the dynamics of an average representative agent. Section 1.2 suggested that the RA equilibrium is a good approximation when the agents have homogeneous propensity to consume, but when agents are heterogeneous the equilibrium predicted by the RA approximation becomes less accurate. In the following computational experiments, households are endowed with identical parameters but they can have different positions in the network. In other words, the heterogeneity of the model lies just in the way households and firms are connected.

This heterogeneity in the connections between agents can be related to the regularity, or symmetry, of the graph. In a star network with one firm in the center and many identical households, the regularity is high and I would expect a good prediction power of the RA equivalent model, as the impact of heterogeneity should be low. Even in the case of many firms that are symmetric stars, employing the same number of households, I would expect the same. Indeed, results show that in these cases the RA equivalent model predicts the equilibrium output without error. However, if the the topology of the graph becomes more irregular, I expect that the RA equilibrium will deviate more from the observed output of the MA model.

Firm's size is used as the conceptual indicator of the regularity/symmetry of the network. If all firms have the same number of employees, the network is perfectly symmetric (the case of multiple stars), while the larger the difference in the number of employees among firms, the more irregular is the network. The variance σ_G of the number of firms' employees is therefore used to measure the regularity of the network.[19]

$$\sigma_G = \frac{1}{|F|} \sum_{f \in F} \left(|H_{Jf}| - \langle |H_{Jf}| \rangle \right)^2 \tag{10}$$

Variance σ_G is equal to zero when the network is perfectly symmetric (multi stars), and it is maximum when $|F| - 1$ firms have just one employee, which is the minimum requirement, and one firm employs the remaining $1 + |H| - |F|$ workers.

As a side note, the idea of studying the impact of symmetry assumptions is used in physics. Following Donoghue et al. [1992], a proposed symmetry may be broken when some interaction does not obey it. The use of symmetry, or average, techniques – like RA models – could still be useful if the interaction that breaks the symmetry is in some sense small. In that case, in a first approximation one can analyze the theory in the limit where the symmetry is valid and then treat the breaking interaction as a perturbation.

Proposition 1. If the population of the MA model is composed just by one household and one firm, if the constraint on household's wealth w_h is not binding (see eq. 4), and if the firm

[19] The main results of the paper are confirmed when using alternative indicators as the Gini or Herfindahl indexes.

is able to produce the expected demand[20] then the MA model coincides with the original RA version of equation 1, with solution 2. In other words, the RA version model can be considered as a special case of the model when $|H| = |F| = 1$ ∎

The aggregate demand can be obtained combining equation 4 with the assumptions that $|H| = |F| = 1$ and that the wealth w_h of the household is not binding (as in the original RA model). Then, the law of motion of output can be derived by introducing the aggregate demand faced by the firm (eq. 5) into equation 7, along with the usual assumption of the Keynesian cross diagram that supply is always able to match demand. Therefore, equation 1 is obtained. Morevoer, all the simulations with $|H| = |F| = 1$ confirm that the MA model behaves exactly as the RA one.

The next step is to explore if and how the deviation Δ_Y from the RA equivalent equilibrium depends on the degree of irregularity of the network, measured by the variance σ_G of the number of firms' employees of randomly generated networks. Figure 3 shows the result for zero-intelligence agents of types 1 and 2, defined according to table 2. The networks are arranged in incremental quantiles, according to their value of σ_G. Therefore, each box on the right always contains more asymmetric network structures with respect to its left neighbor. In the leftmost box there are only cases of perfect symmetry, i.e. $\sigma_G = 0$, for reasons that are clarified in the following proposition.

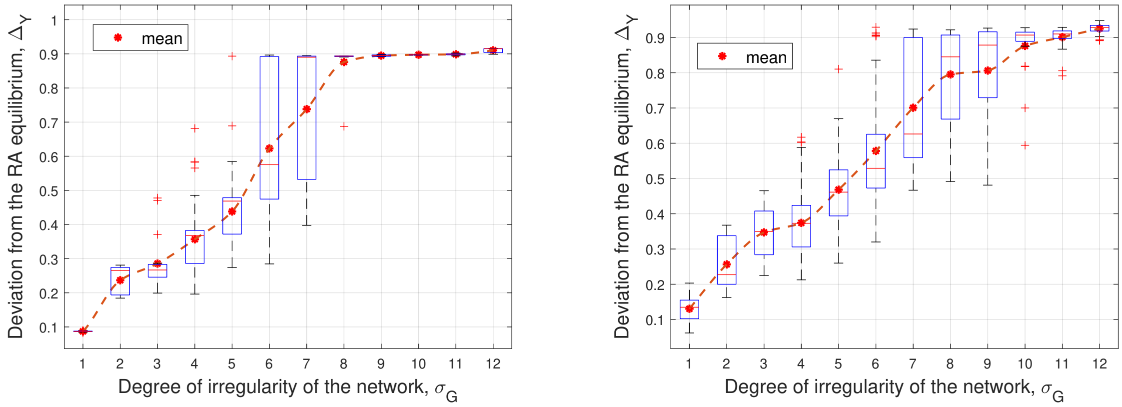
Proposition 2. For every non zero intelligence definition of household's rationality (see table 2): $\sigma_G = 0 \implies \Delta_Y = 0$. In other words, if the network structure is perfectly symmetric ($\sigma_G = 0$), the deviation Δ_Y from the RA equivalent equilibrium is equal to zero ∎

In the leftmost boxes of figures 4a and 4b, characterized by $\sigma_G = 0$, the median and mean deviation from the RA equilibrium are always zero ($\Delta_Y = 0$).[21] This proposition tells that the equilibrium output of the RA equivalent is a perfect approximation of the network model when it is completely symmetric, that is, when all firms have the same size, and therefore the same potential production. The rationale for this result is that in a scenario where all firms have the same size, and are connected to consumers in a symmetric way (meaning that each firm has the same number of households connected by consumption links), the production of each firm can be evenly split among households, without any rationing of consumers. Therefore, it exists a simple configuration where all firms sell to the same number of households, which guarantees the achievement of the RA equilibrium without any rationing of households, who purchase the same fraction of production. The basic example is having only one firm, which sells the same fraction of production to its identical customers. In this case there are no problems of aggregation and no reason to deviate from the RA equilibrium. The same holds for $|F|$ identical firms, which at the equilibrium will have the same number of customers,[22] purchasing the same quantity of goods. This result does not hold for zero intelligence households (see figure 3) because they are not able to properly redirect their consumption links, reaching the correct symmetric configuration, either because they are too static (fixed links case), not reacting at all when rationed in consumption, or too dynamic (random links case), reacting also when they should not. In any case, when $\sigma_G = 0$, the deviation Δ_Y is very small also for zero intelligence households.

[20] To produce the expected (past) demand, the term $\beta \cdot (1 + extra) \cdot |H_{Jf}|$ of eq. 7 should not be binding. It is worth noting that the aggregate Keynesian cross diagram of eq. 1 implicitly satisfies all these conditions.

[21] The visual perception of figure 4 is confirmed by the numerical outcomes of the computational experiments, showing that $\sigma_G = 0 \implies \Delta_Y = 0$.

[22] Consumption links for each household are $\frac{|F|}{2}$ (see table 2).



(a) Random links (b) Fixed links

Figure 3: Deviation from the RA equilibrium Δ_Y as a function of the irregularity of the network σ_G. The cases of zero-intelligence agents of type 1 (random links) and 2 (fixed links) are presented.

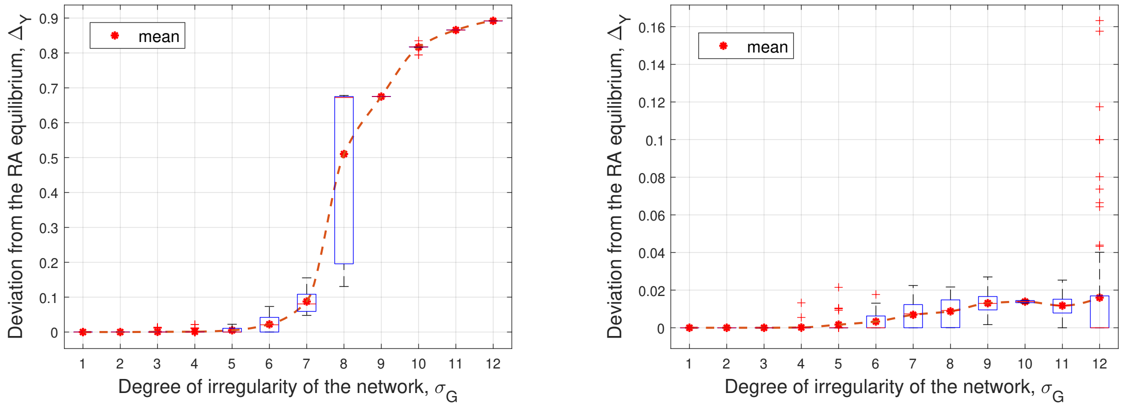
Proposition 3. When the rationality of the households is low, the higher the regularity of the network, the smaller the deviation from the RA equilibrium.

The proposition is validated by a visual inspection of figures 3 and 4a. This is one of the key results of the paper, especially if examined with **Proposition 4**. It shows that, even when households are homogeneous, i.e. endowed with the same parameter set, the prediction power of the RA equivalent model is poorer when the topology of the network is less regular. The main reason lies in the inefficient allocation of consumption across firms by households, due to their low rationality combined with the asymmetry of the network. If there are big and small firms in the economy, households are not able to allocate correctly their consumption budget among the exiting firms. This result acquires more relevance if compared to **Proposition 4**, and will therefore be discussed after presenting it.

Proposition 4. When the rationality of the households is high, the deviation from the RA equilibrium is low for every level of regularity of the network.

The proposition is proved by a visual inspection of figure 4b, which shows the deviation from the RA equilibrium as a function of the irregularity of the network in the case of semi-rational agents. This deviation is very small (always less than 2%), irrespective of the regularity of the network. The last two results show that the RA model is able to approximate the equilibrium output when the network of interactions is regular or when the households are rational enough. In the case neither of these conditions is fulfilled, the RA equilibrium becomes inaccurate.

The deviation from the RA equilibrium Δ_Y can also be considered as a sort of output gap, where the RA equilibrium output is the potential output, and the equilibrium of the MA model is always lower, due to the frictions related to coordination failures in the network structure. In this perspective, low rationality household need a very regular market structure in order to allocate demand properly and to get close to the potential output. On the other hand, rational agents do not need a regular structure because they are able to adapt to the market structure, even if it is irregular, therefore reaching the potential output. This feature resembles the argument in Gode and Sunder [1993], where allocative efficiency depends on the market structure, which is therefore considered as a "partial substitute for individual rationality". In



(a) Bounded rational

(b) Semi rational

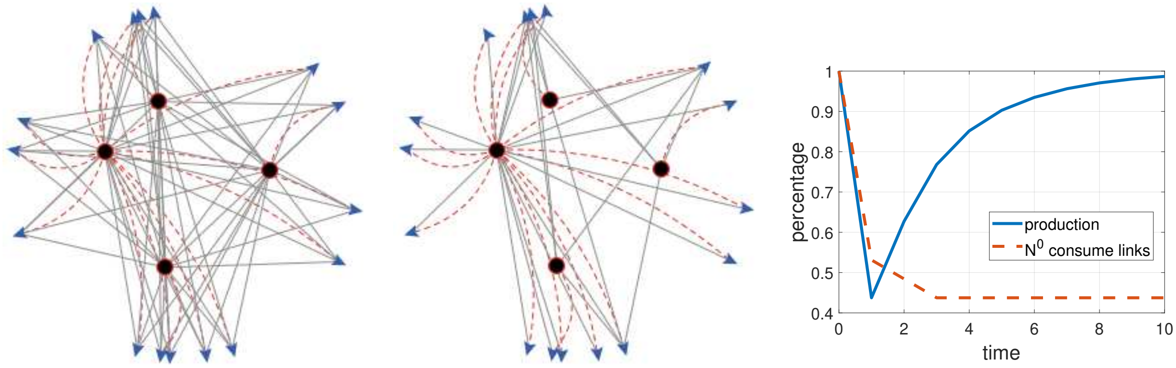
Figure 4: Deviation Δ_Y from the RA equilibrium as a function of the irregularity of the network σ_G. The cases of bounded rational and semi rational agents are presented.

this work, a regular structure plays the role of a partial substitute for individual rationality.

In order to understand the dynamics leading to this analogy between regularity and rationality, let me compare the case of semi-rational agents with the bounded rational one. According to table 2, the only difference between these two cases is that bounded rational households always try to increase their consumption links[23], whereas the semi-rational households do it only if they have been globally rationed, which means they consumed a lower quantity of goods with respect to their desired level. When the network is asymmetric, some firms can produce more (the ones with more workers) and some others less. Therefore, rational households should purchase more from bigger companies. Bounded rational households, facing firms with heterogeneous size, cut their consumption link if they are rationed by a (presumably) small firm but then they try to connect again to another firm, which could be small again, leading to a new possible rationing of the household. Differently, semi-rational agents do not try to connect again if they are not rationed, meaning that they are happy if consuming all there consumption budget from one firm or just from a few ones. In a sense, the small additional rationality of the semi-rational households allow them to adapt to the network structure, and from a more aggregate point of view, allows the whole network structure of the consumption links to adapt in order to counterbalance the asymmetry of the job links structure.

Figure 5 shows an example of this links' dynamics in a network composed by four companies and sixteen semi-rational households, who consume initially from all firms. The irregularity of the network is maximum because one firm employs thirteen workers, while the last three firms have one worker each. Box 5c shows that the output of the economy initially collapses, due to the "coordination failure" of households, which consume too much from smaller firms and not enough from larger ones. However, over time, semi-rational agents cut their useless links with small firms, focusing on larger firms, and the aggregate output grows again towards the RA equilibrium. As it can be observed by comparing figures 5a and 5b, the network adapted, and the final structure of the consumption links counterbalances, or neutralizes, the initial asymmetry of the network.

[23]To be precise, they do it only if their current number of consumption links is lower with respect the initial one.



(a) Initial graph (b) Final graph (c) Dynamics

Figure 5: Sample model with 16 households (triangles) and 4 asymmetric firms (circles). Households cut consumption links (straight lines) if rationed. Production, after an initial collapse, converges to the RA equilibrium.

2.1.1 Rationality of firms: the labor market

This section hosts a short digression with respect to the main setting of the paper, where I show that similar results are obtained if firms, instead of households, are assumed to be more rational. In fact, having companies that do not hire or fire workers (in other words, assuming that firms behave according a low rationality level) means imposing a strong rigidity to the economic system. If this assumption is relaxed, introducing a rudimentary labour market, firms will be able to modify the number of employees according to some economic feedback.

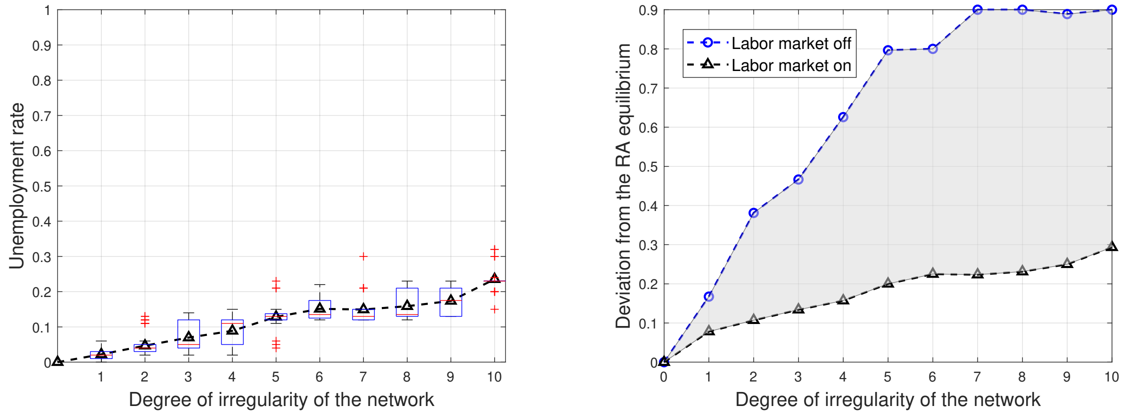
A very simple decision making of firms is then designed. A company computes its excess of workers by comparing current workers to the needed workers to produce the faced demand, as in equation 11:

$$\Delta|H_{Jf}|(t) = \text{int}\left(\frac{\beta \cdot |H_{Jf}(t)| - z_f(t)}{\beta}\right). \tag{11}$$

Firms with an excess of workers ($\Delta|H_{Jf}|(t) > 0$) will fire them, whereas firms facing a lack of workers ($\Delta|H_{Jf}|(t) > 0$) will hire the correspondent number of workers, if available. Assuming that the goods market (consumption) goes on a weekly basis, the labor market goes on a monthly basis, i.e. firms modify their labour force every four consumption steps.[24]

Results show that endowing firms with higher rationality has similar effects to those found for households. When the network structure becomes irregular, zero intelligence households are not able to allocate consumption correctly and GDP quickly decreases, as shown in the upper line of figure 6b. On the other hand, when the labor market is at work, firms are able to adjust the labour force according to adaptive expectations, and the performance of the economy improved considerably (see the lower line of figure 6b). The shaded area becomes wider when the asymmetry of the network increases, showing how the presence of the labor market allows firms to counterbalance the asymmetry of the economic system, in a way that resembles the one explained in figure 5, where the consumption links reallocation is here replaced by a reallocation of the job links. In any case, the irregularity of the network still conditions the performance of the economy, as shown by figure 6a. When the initial arrangement of the job links is irregular, firms try correct it (and succeed, observing the shaded area of figure 6b) but lose workers, demand and production in the process, reaching higher levels of unemployment rate.

[24]This choice is mainly an aesthetic detail, but results are robust with respect to the relative frequency of the two markets. Of course, lower labour market frequency means lower overall efficiency.



(a) Unemployment rate (b) Labour/no-labour GDP gap

Figure 6: Unemployment rate (on the left) and the deviation Δ_Y from the RA equilibrium (on the right) as a function of the irregularity of the network σ_G. The shaded area represents the gap between the model with or without labour market. The cases of zero-intelligence fixed links agents is presented.

2.1.2 On the absence of prices

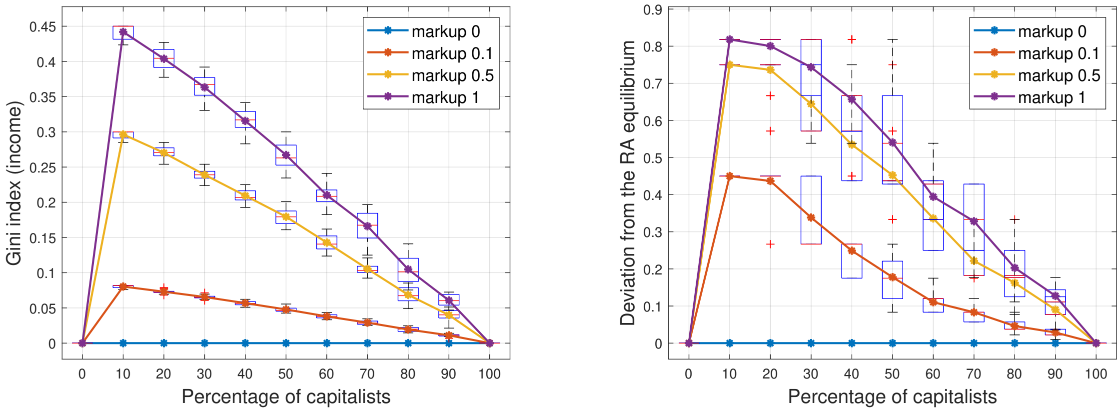
Actually, what I called "labor market" simply consists in determining the demand of workers by firms, being the supply composed by all the available (unemployed) households. The cost of labor is not explicitly considered, as the company just distributes all its revenues to the workers. Hence, there is no role for wage as a coordination mechanism. The same can be said about the price of the consumption good. Firms cannot adjust prices, and therefore an important instrument that can improve the efficiency of the economy is missing. In the perspective of this work, prices can be seen as an additional form of rationality that is present in the economic system. If companies would be endowed with pricing decision rationality, the economy would benefit of an additional mechanism for smoothing out the complexity of the system, in a way that is conceptually not too different from the other forms of rationality introduced in the paper. Although pricing mechanisms might be designed in the model, I think this would lead too far away with respect to the aim of this paper, which is technically a simple extension of the Keynesian cross diagram, inheriting all the main properties from the original model, including the absence of prices.

2.2 Inequality

In the previous section it has been shown that rationality is able to "correct" the asymmetries in the economic network, and therefore, if agents are endowed with enough rationality, the RA model provides a good approximation of the equilibrium output.

What happens, however, if the asymmetry is not limited to the job links geometry but also affects the income that flows through them? In other words, what happens if income inequality is considered?

Income inequality is introduced in the model by differentiating two categories of income that flows from firms to households: wage l_h and dividends d_h. All households work in the firm, but normal workers receive only the wage, whereas shareholders, or capitalists, receive also the dividends. Thus, each firm f pays a part of its revenue as wages l_f, and distributes the rest (profits) to shareholders, as dividends d_f. In order to stick to a textbook representation



(a) Gini index (income)

(b) Δ_Y

Figure 7: Δ_Y and Gini index with respect to the percentage of capitalists in the case of semi-rational agents and symmetric network.

of the labor market, see Blanchard [2017] for details,[25], I introduce the concept of markup μ as an indicator of the share of revenues that are paid to the firms, and then distributed to shareholders. Then, given a markup μ, and assuming that all firms have the same markup, the wage bill and profits (paid as dividends) of firm f are:

$$\textbf{wage bill: } l_f(t+1) = \frac{r_f(t)}{1+\mu}; \qquad \textbf{profits: } d_f(t+1) = \frac{\mu \cdot r_f(t)}{1+\mu}. \tag{12}$$

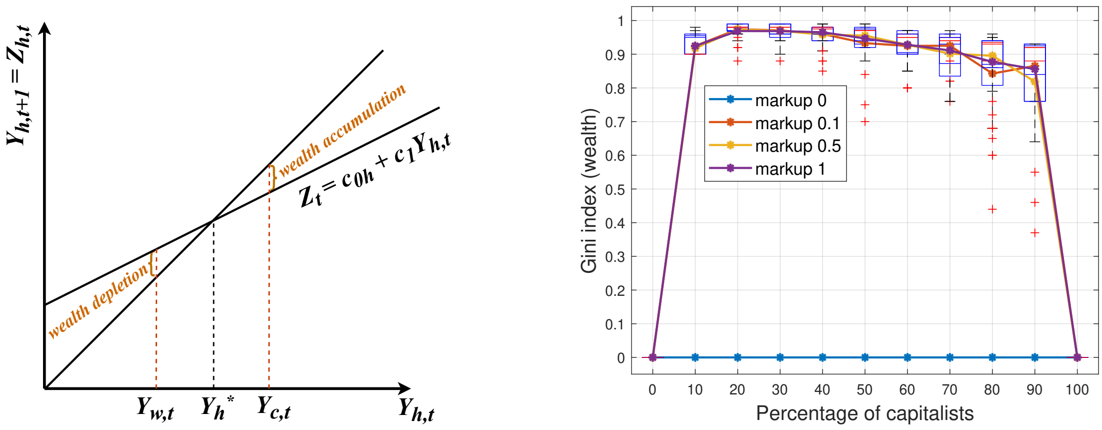
This is of course a stylized description, which simply remands to the fact that for higher markup the level of competition in the market is lower and there is more room for profit. The implications and limitations of this framework will be discussed below, when results are presented and commented. Its relevance in this study is mainly related to the possibility of exploring different levels of concentration of income in the economy by varying the markup and the percentage of capitalists in each firm.

For the sake of simplicity, I make a few additional assumptions, which can be easily relaxed, and which only marginally affect the core results of this section: (i) the shareholders of a firm f are also workers of firm f, (ii) the markup and the percentage of shareholders are static and equal for all firms, (iii) the percentage of shareholders can vary from one worker per firm to all workers of the firm, thus the minimum number of shareholders is equal to $|F|$, (iv) households are semi-rational, which is the highest rationality level considered in the paper, leading to a negligible RA deviation in the previous section (see **Proposition 4**).

Proposition 5. In the presence of capitalists, which receive income in form of dividends, there is a significant deviation from the RA equilibrium. This deviation is larger when the markup is high and when the percentage of capitalists is low.

This result is validated by a visual inspection of figure 7, which present the case of an asymmetric network ($\sigma_G \neq 0$). In particular, the left box of the figure shows how the percentage of capitalists and the markup affect income inequality (measured by the Gini index), while the right box shows how they affect the deviation from the RA equilibrium Δ_Y.

[25]The labor market is then combined in this widespread textbooks with the IS-LM model to obtain the AS-AD or IS-LM-PC models.



(a) Wealth dynamics

(b) Wealth inequality

Figure 8: Wealth dynamics fo **c**apitalists and **w**orkers, and wealth inequality at the equilibrium in a perfectly symmetric network.

For a 0% or 100% percentage of capitalists there is no income inequality and no deviation from the RA equilibrium. In fact, these two cases are completely overlapping because 100% of capitalists means that dividends are distributed as income to all households.

A big jump is clearly visible from 0% to 10%, corresponding to the case of one capitalist per firm, which is the minimum number of capitalists in the model. In the 10% case, income inequality is the largest and the prediction of the RA equilibrium is the weakest. Going below the 10% level of capitalists would imply that some firms have no capitalists among their workers, and that some households are shareholders of many firms. In this case inequality and deviation from the RA equilibrium would be even more pronounced. When the percentage of capitalists increase, both income inequality and deviation from the RA equilibrium decline, until the zero inequality case of 100% capitalist is reached.

Proposition 2 holds if the network is perfectly symmetric and there is no inequality. However, when there is inequality, Proposition 2 does not hold any more, and the deviation from the RA equilibrium is far from being zero. So, even if household's parameter are homogeneous, and the network is symmetric, the equilibrium output is lower than the RA prediction, or potential output. The reason is that capitalists earn a high income ($Y_{c,t}$) and they have a target consumption that is lower than their income, as shown in figure 8a. As a consequence, they accumulate wealth, which is not used to feed demand (see figure 8b). On the other hand, the demand of the low-income, or non-capitalist, workers is higher than their income $Y_{w,t}$ and they need to use part of their wealth in order to consume the desired amount. Thus, they consume their wealth until it is exhausted, becoming finally unable to purchase the target amount of consumption goods. This decrease in consumption of low-income workers reduces firms' sales, revenues, and future production, leading to a lower equilibrium output.

Figure 8b reveals the strong inequality in wealth distribution, caused by the accumulation of wealth by capitalists. The accumulated wealth is fed by "excess savings" out of capitalists' income, which do not contribute to the aggregate demand any more, but remain unused in the economy. It should be remarked that the model does not take into account potential mechanisms that could mitigate the unproductive accumulation of wealth by capitalists. Discussing these mechanisms goes beyond the scope of this paper, but I will spend a few words on investments and income redistribution, which are among the main ones. Purchasing investment goods is a common way to spend profits, feeding demand. This would mitigate the contraction of the

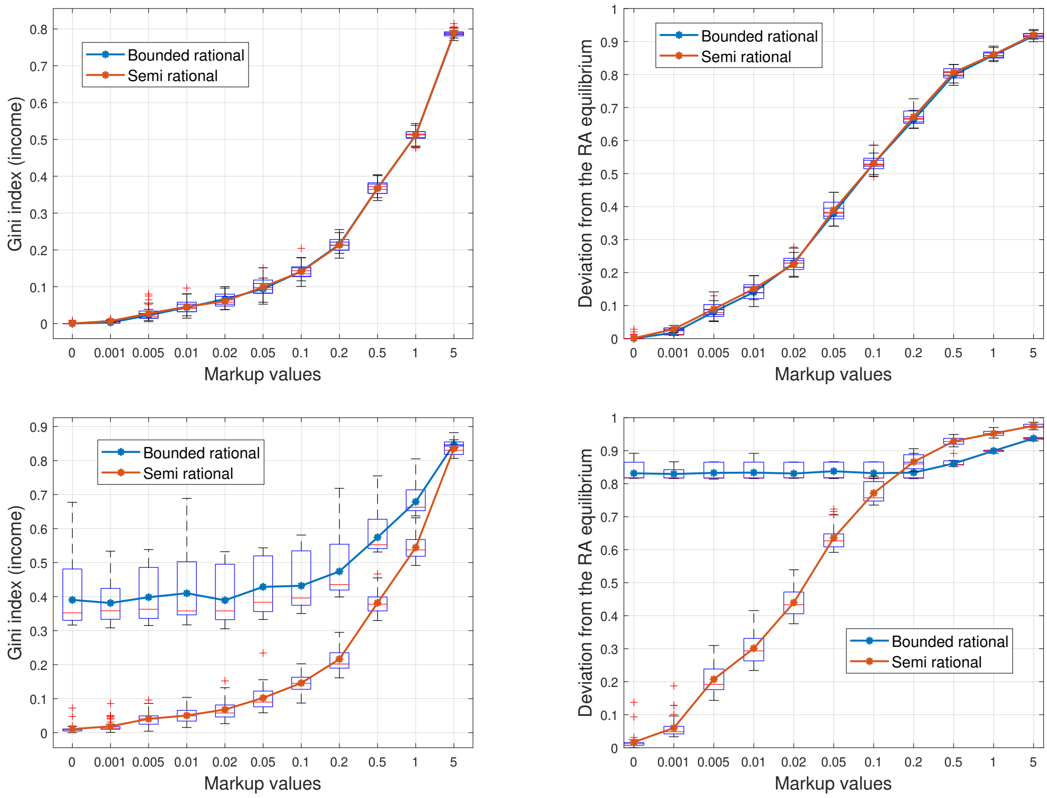
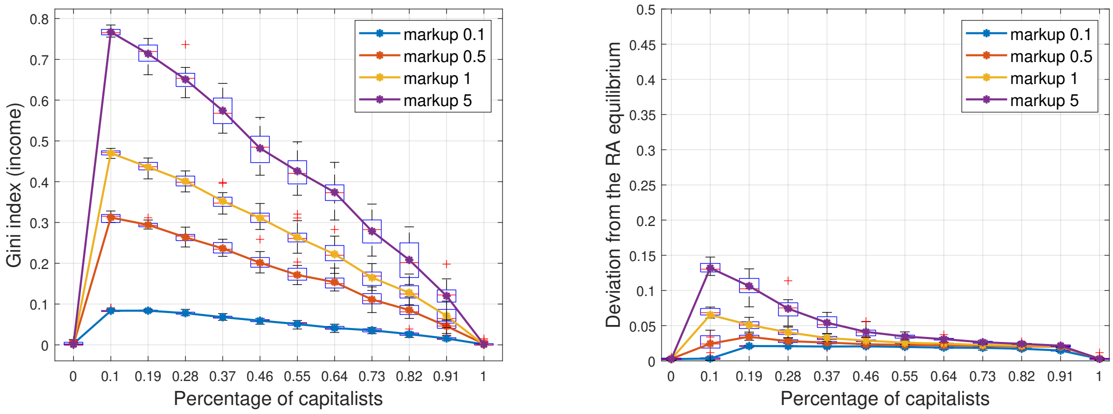


Figure 9: Income Gini index and Δ_Y vs. markup in regular (up) and irregular (down) networks.

equilibrium output, but still does not solve the problem of low demand of consumption goods, as investments in physical capital or R&D tend to reinforce the supply side of the market, while the emerging problem here is a contraction of households' demand. The second mechanism is the redistribution of income (or wealth), which can be carried out by taxation or by the credit/financial markets. In section 2.3 the effect of an income tax will be examined.

Going back to figure 8a, note that the demand of each household h at time t is given by $z_h(t) = \min\left[c_{0h} + c_1 y_h(t), w_h(t)\right]$ (eq. 4). When $z_h(t) - y_h(t) < 0$, the household is consuming less than its income, accumulating wealth. Equation 4 shows that this happens when $y_h(t) > \frac{c_{0h}}{1-c_{1h}}$. In particular, if parameters are homogeneous across households, i.e. $c_{0h} = \frac{c_0}{|H|}$, and $c_{1h} = c_1$, it happens when $y_h(t) > y_{RA}$, where y_{RA} is the RA equilibrium income per capita. Therefore, each household with a higher than average income is accumulating wealth, as $\Delta w(t) = y_h(t) - c_h(t) > 0$, with consumption $c_h(t) \leq z_h(t)$, due to potential rationing. Similarly, it can be shown that each low income household depletes wealth in order to match a desired consumption that is above its income. The household will before or later exhaust its wealth, being able to consume just out of its income, therefore reducing aggregate demand and production.

Figure 9 shows the Gini index of income and the deviation from the RA equilibrium for different level of inequality, measured by the markup. The percentage of capitalists is fixed to 10% of households. There are some non trivial interactions between rationality, regularity and inequality, which are pointed out by the figure. First, when the network is regular, i.e., firms have similar size, the economic outcomes for semi rational or bounded rational households are almost indistinguishable. This is due to the fact that rationality is not needed to behave correctly in a simple world. However, when the network is not regular, a significant difference between higher or lower rationality emerges. For low markup values, more rational agents are



(a) Gini index (income)

(b) Δ_Y

Figure 10: Δ_Y and Gini index with respect to the percentage of capitalists in the case of semi-rational agents and symmetric network.

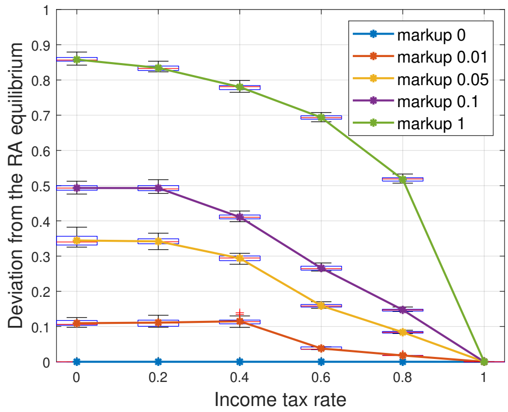
able to coordinate and reach a correct allocation of the consumption budget, which leads to lower inequality[26] and higher output. Conversely, for very high markup values, when income is unequally distributed and the excess saving of capitalists is very high (see figure 8a), the economy populated by more rational households achieves a slightly lower output. The interpretation of this puzzling outcome might be the following: when the economic system shows structural inequality and lack of consumption from the high income households, a rational allocation of consumption might converge more efficiently to the "bad equilibrium", while the presence of bounded rationality introduces some noise that prevents to reach the bad equilibrium smoothly. To explain it with a simile, if a team has a very bad trainer, coordinating rationally on his bad strategies might even lead to worst results with respect to acting more randomly.

As a finale note, the strong output reduction in the case of income inequality depends on the assumption of homogeneous parameters for households. In particular, the autonomous consumption c_{0h}, which is homogeneous over households, is intended as the minimum level, irrespective of income, which households want to consume in any case; a sort of survival consumption. However, if we assume that also autonomous consumption is proportional to the share of household's income $c_{0h}(t) = c_0 \frac{y_h(t)}{Y(t)}$, *de facto* negating any role for c_{0h}, the effect of inequality in output would be greatly reduced, as shown in figure 10. In this case, consumption becomes proportional to income and capitalists increase their consumer spending, thus increasing also aggregate demand and output. Interestingly, for low percentages of capitalists, there is still a significant deviation from the RA equilibrium, showing a relevant interaction between the presence of a network structure and inequality.

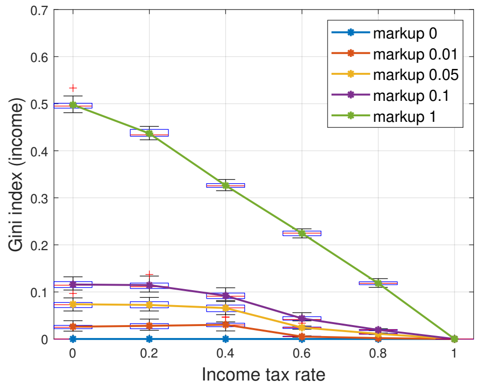
2.3 Redistribution

Results of the previous section show that income concentration reduces GDP. The main reason is that low-income households have not enough resources to purchase the desired amount of goods, therefore reducing demand and production. This lack of aggregate demand can be addressed in the model by designing redistribution mechanisms that allow low-income households to match their desired consumption level. The two main instruments that can serve this purpose are

[26]The income inequality for bounded rational is explained by lack of coordination that leads to the existence of larger firms that do not sell enough goods and that, therefore, pay a lower salary.



(a) Gini index (income)



(b) Δ_Y

Figure 11: Δ_Y and Gini index for different values of the income tax in the case of bounded rational agents and irregular network.

taxation or credit; in both cases purchasing power would be transferred from whom owns it in excess to whom lacks it.

In order to host a credit sector, the model should be substantially enriched, as many essential ingredients are missing, e.g. economic growth, money, and inflation. In particular, the absence of income growth prevents households to repay any debt with positive, or even zero, interest rate. Households would actually use credit to reach their desired consumption level, but without income growth they will never be able to save in order to repay it. On the other hand, it is easy to design a public sector in charge of fiscal policy. For the sake of simplicity, I assume a zero budget target for the government, which collects taxes and transfers them back to households. Among the many possible fiscal architectures, I show here the example of a government that collects taxes according to a progressive tax system, with fixed tax rate τ and a tax deduction equal to half the average income. The government uses all the collected taxes to pay a universal basic income, which is equal for every household.

Results are shown in figure 11, where the Gini index of income and the output gap are plotted as a function of the tax rate. The higher the tax rate on income, the higher the redistribution of income from wealthy households to poor households, and the better the outcome of the economy in termes of output and inequality. It is worth noting that the redistribution policy is effective even if it starts later in time, when wealth is already in the hands of the few capitalists. This happens because capitalists, with a lower after-tax income, need to consume out of their wealth, thus decreasing it, whereas workers have enough income to match their desired consumption and to increase the level of aggregate demand.

Conclusions

This study presents an extension of the Keynesian cross diagram, where the implicit hypothesis of representative agents is relaxed and the aggregate demand is the sum of local demands of multiple households. Firms and households are connected through job links and consumption links, according to network structures that can have different degrees of symmetry (or regularity). Households are endowed with several levels of rationality, which reflect their capacity of allocating the consumption budget across various firms. The model collapses into the classic Keynesian cross diagram when the number of households and firms is equal to one.

The study sheds light on the conceptual relation between agents' rationality and the regularity of economic interactions, which can be considered as an approximation of world's complexity. If the world is complex (asymmetric network) and households are not rational enough, the GDP is lower than the potential output, which is the one predicted by the representative agent model, and corresponds to the perfect allocation of the consumption budget. If the world is less complex, or if the agents are more rational, the GDP is closer to the potential output. A clear and natural trade-off emerges from our study, between the complexity of economic choices, which is represented by the asymmetry of the network structure, and the rationality of the agents, which corresponds to their capacity to make correct choices in a complex setting. Under this respect our study resounds with the idea of Gode and Sunder [1993] about markets as partial substitutes of individual rationality. Moreover, results show that the representative agent assumption, which ignores coordination failures, becomes a poor approximation when the combination of bounded rationality and system's complexity matters.

The second main result of the paper concerns the impact of income inequality on GDP. Income heterogeneity has been incorporated in the model by designating a part of the households as capitalists, who receive dividends from companies. The model suggests that GDP declines when income inequality is higher. This is due to the wealth accumulation process of high income households, who consume a smaller fraction of their income, even if the propensity to consume is equal for all. Symmetrically, low income households are not able to match their desired consumption because they become eventually wealth constrained. On the whole, aggregated demand becomes lower. A redistribution mechanism, here implemented by means of a government that collects a progressive tax and pays a universal basic income, reduces inequality and restores higher GDP levels.

Results should be read with a grain of salt. The presented models are the extension of an abstract framework, and the contribution of the paper should be considered under a conceptual perspective and not under an operational one. In the real world, the notion of complexity is not limited to firms' size heterogeneity or to consumption budget allocation problems. On the other hand, the rationality of human beings can be more powerful than the mere ability to solve a basic allocation problem. However, this paper is able to quantify, in a simple and specific case, the cost of the RA assumption in terms of prediction error.

As in the original Keynesian cross diagram, the models are demand driven and do not consider the relation between capitalists' accumulated wealth and investments, which can affect both the demand and the supply sides of the economy. Nevertheless, the MA extensions of the Keynesian cross diagram provide a description of the effects of income inequality on output, which the original model is not even able to conceive. Furthermore, even if capitalists' wealth can be reinvested, it is hard to imagine how it could solve the problem of the lack of consumption goods demand by low income households, at least in a model that does not envisage income growth. Capitalists' excess savings might be available for consumption of low income households at some cost, but sooner or later these households will face again wealth constraints, unless their income growth is large enough to repay the debt. These type of problems may be addressed in future developments of the model.

It might seem excessive to derive policy implications from the model, as it does not rigorously reflect the functioning of real economies. Nevertheless, I think that a few basic conclusion can be drawn: (i) aggregate economic models underestimate the effects of coordination failures on aggregate demand, (ii) if the standard rationality assumptions – homo economicus – are relaxed, the reliability of aggregate models decreases, (iii) income inequality generates poor economic results, especially if coupled with bounded rationality (vi) perfect rationality alone is not a valid defense against the negative impact of inequality on aggregate demand and output, (v) fiscal policy is not only useful to recover from recessions, but also to redistribute income and, thus, foster demand, (vi) excess savings of high income households can be detrimental to the economy,

(vii) if the economy faces a negative demand shock, as the current one caused by Covid-19, where the consumption of many goods and services sharply decreased (e.g. travel, hospitality, art and entertainment, etc), along with the wage bill of mainly low income households, excess savings of high income households might increase and become specially harmful, as they barely flow back into the economy, thus (viii) redistributive fiscal policies seem to be particularly relevant during recessions that amplify the "saving gap" between high and low income households.

Finally, this paper aims also at moving RA and MA models closer, showing that MA models may represent useful extensions of RA models. There is some potential for models based on interacting agents, which has been largely ignored in the past. I think that recent events with relevant economic impact, e.g., the last financial crisis, the climate change phenomenon, and the spread of Covid-19, confirm the convenience to draw attention to the enrichment of economic models in this direction. Actually, the same methodology that has been applied to the Keynesian cross diagram in this paper could be applied to other representative agent models, and the exercise might prove to be an interesting one, revealing some limitations of the original approach, and suggesting new paths for future research.

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